Comments on the distinction between color- and flavor-branes and new D3-D7 solutions with eight supercharges

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Abstract

We investigate the distinction between color- and flavor-branes that is usually made in the context of gauge/string duality with backreacting flavors. Our remarks are based on a series of examples concerning the role of source terms in relatively simple supergravity backgrounds that allow for a well-controlled approach to the problem. The observations suggest that, in opposite to general practice, one could consider such terms for both kinds of branes, while their presence is only essential for smeared sources – as is commonly the case for flavor-branes.

Among the examples studied are D3-D7 systems with eight supercharges, where the D7-branes are assumed to be smeared. Starting from a fairly generic ansatz, we will find new analytic and numeric solutions and briefly compare these to previous work in this field.

1 Introduction

In the context of gauge/string duality, recent years have seen the adoption of a standard method when it comes to the study of gauge theories with a large number of flavors in the Veneziano limit using supergravity- and brane-actions. Starting with [1], [2] and reviewed in [3], the methodology is founded on the addition of further branes to the background, that are space-time filling while also extending along non-compact transverse cycles.

Prior to flavoring, the supergravity background is always thought of as the near-horizon geometry of a stack of branes that might wrap a compact cycle [4] or be placed at the tip of a singular manifold [5], but is in each case described by the equations of motion of the suitable ten- or eleven-dimensional supergravity. That is, there is an action $S_{\mathrm{IIA/B}}$ or S_{M} that gives rise to the relevant equations of motion solved by the background in question, which is argued to be dual to a certain gauge theory.

Now, to add flavors to the system, one adds to this supergravity action the action of the new flavor-branes S_{flavor}, and solves the equations of motion of the

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system

$$S = S_{\mathrm{M,IIA/B}} + S_{\mathrm{flavor}} \tag{1}$$

S_{flavor} is a superposition of N_f D-brane actions consisting of the standard DBI- and Wess-Zumino terms. There is a multitude of arguments supporting this procedure, some of which we will briefly mention here. See [3] for a more thorough discussion. From the point of view of the 't Hooft expansion of the gauge theory, flavor should lead to diagrams with boundaries, corresponding to an open string sector in the dual string theory. And as it is well known, one adds an open string sector to a type II theory by including D-branes. Apart from the fact that only the flavor branes appear in the action (1) there are further conceptual differences between the color and flavor-branes in the background. Flavor is a global symmetry while color is related to a local, gauged one. Hence flavor charged objects exist in non singlet states, while this is only possible for color-charged ones in non-confining phases.

The difference between the two kinds of branes mentioned is reflected in (1) – there is a source term for the flavor branes, while the physics of the color-branes is captured by the supergravity action. From the point of view of gauge/string duality, the physics of the pure Yang-Mills sector (e.g. glueballs) are captured by the supergravity action, those of the open strings describing the fundamental matter (mesons) by the brane action and interactions between the two by the fact that the background fields as well as world-volume fields couple in the brane action. In this paper, we will critically investigate this statement, the form of (1) and the distinction between flavor- and color-branes. Working in the supergravity limit, our observations will be based on a series of examples signifying the relevance of source-terms such as S_{flavor} in (1) for various brane-solutions.

In section 2 we will recall some known yet often overlooked[1] facts about source terms for p-branes in supergravity – we are thinking here of the standard 1/2-BPS brane solutions of type IIA/B and M-theory. These are usually found by studying the equations of motion of a supergravity action $S_{\mathrm{M,IIA/B}}$. Yet as we will remind the reader, the p-brane solutions solve these equations of motion only if one excises the locations of the branes from space-time. Upon adding a suitable source term to the equations of motion the equations of motion are solved everywhere. From the point of view of finding solutions to the equations of motion, one can ignore the source term as the sources are not distributed over an open subset of space-time; they can be smeared along some directions, as long as they are localized in others.

This changes in section 3 where we will see that source terms are essential once we start to smear the branes over open subsets. The crucial point is that sources generally lead to the violation of the Bianchi identity of a magnetic field strength, e.g. $\mathrm{d}F_{(D-p-2)} = \rho_{(D-p-1)}$. Only as long as the branes are localized can one ignore the source-density ρ and proceed by working with the supergravity action alone.[2]

These observations of sections 2 and 3 imply that in the generic case (1) implicitly contains a source-term for the color-branes and should be replaced

[1] See however [6] and [7] which, working in the boundary state formalism, do include source terms for localized color- and flavor-branes.

[2] We do simplify things here, as Bianchi identities can also be violated by the presence of Chern-Simons terms. In most of the examples we have in mind however, this is not the case.

with

$$S = S_{\mathrm{M,IIA/B}} + S_{\mathrm{color}} + S_{\mathrm{flavor}} \tag{2}$$

With this in mind, we will turn in section 4 to the relatively simple problem of D3-D7 systems with eight supercharges, corresponding to duals of $\mathcal{N} = 2$ gauge theories in $d = 3 + 1$ dimensions. Here we will compare the backgrounds found by [8] - [10], which solve the pure supergravity equations of motion without any source terms for either flavor or color-branes, to new solutions, found working in the spirit of [2] solving the system given by $S_{\mathrm{IIB}} + S_{\mathrm{flavor}}$. As we will see, the two approaches lead to a set of equations and solutions which only differ by the fact that our new ansatz implies that the D7-branes are smeared. From this it can be implied that the two approaches are equivalent. Moreover, we will see that a series of T-dualities can exchange the color- and flavor-branes, although we did include source terms for the latter and not for the former. From all these observations we conclude that equation (2) is the most suitable ansatz, leading us to the one of the main observation of this paper: From a technical point of view, source terms can be ignored as long as the sources are localized, yet when using (1) to make statements about the dynamics of the resulting system, care has to be taken.

Finally, we use the results of section 4 to find some new analytic (numeric) solutions to the D3-D7 system in section 4.3 (4.5). Note that many of the results presented in section 2 are not new, but seem to be often overlooked in this line of research.

2 p-brane sources

As a warm-up, we will review source terms for p-brane solutions (see [11]). The equations of motion derived from[3]

$$S_{\mathrm{grav}} = \frac{1}{16\pi G_D}\int [\mathrm{d}^D x\sqrt{-g}(R - \frac{1}{2}\partial^\mu\Phi\partial_\mu\Phi) - \frac{1}{2}e^{a\Phi}F_{(p+2)}\wedge *F_{(p+2)}] \tag{3}$$

are solved by

$$\begin{aligned} \mathrm{d}s^2 &= H_d(y)^{-2\frac{d-2}{\Delta}}\mathrm{d}x_{1,p}^2 + H_d(y)^{2\frac{p+1}{\Delta}}\mathrm{d}y_d^2 \\ F_{(p+2)} &= e^{-\frac{a}{2}\Phi_\infty}\sqrt{2\frac{D-2}{\Delta}}\mathrm{d}(H_d(y)^{-1} - 1)\wedge \mathrm{d}x^0\wedge\cdots\wedge \mathrm{d}x^p \\ e^\Phi &= e^{\Phi_\infty}H_d(y)^{a\frac{D-2}{\Delta}} \end{aligned} \tag{4}$$

with Φ_∞ constant, $D = (p+1) + d$, $y^2 = y^a y^a$ and

$$\begin{aligned} \Delta &= (p+1)(d-2) + \frac{1}{2}a^2(D-2) \\ H_d &= 1 + \left\{ \begin{array}{cc} h_1|y| & d = 1 \\ h_2\log y & d = 2 \\ \frac{h_d}{y^{d-2}} & d \geq 3 \end{array} \right. \end{aligned} \tag{5}$$

[3] As we mentioned in the introduction, our ansatz here excludes the possibility of Chern-Simons terms as are relevant for [12] for example.

the standard 1/2-BPS p-brane solutions of supergravity. The solutions are interpeted as describing branes at $y = 0$. However, (4) do not solve the equations of motion *everywhere*, as $H_d(y)$ satisfies

$$\Box_{\mathbb{R}^d} H_d = -\frac{d(d-2)\pi^{d/2} h_d}{\Gamma(d/2)} \delta^{(d)}(y) \tag{6}$$

and the equations of motion do contain the d-dimensional Laplacian $\Box_{\mathbb{R}^d}$. These singularities are of course due to the p-brane at the origin and can be lifted by adding a source term to the action, $S = S_{\text{grav}} + S_{\text{src}}$,

$$S_{\text{src}} = -\frac{T_p}{2} \int \mathrm{d}^{p+1}\xi \sqrt{-\gamma} e^{b\Phi} [\gamma^{ij} \partial_i X^\mu \partial_j X^\nu g_{\mu\nu} + (p-1)] + \mu_p \int X^* C_{(p+1)} \tag{7}$$

S_{src} introduces additional terms to the equations of motion.[4] E.g. in the case of the Maxwell equation

$$\begin{aligned} 0 &= \partial_\mu (\sqrt{-g} e^{a\Phi} F^{\mu\nu_0 \dots \nu_p}) \\ &\quad + 16\pi G_D \mu_p \int \mathrm{d}^{p+1}\xi \epsilon^{i_0 \dots i_p} \partial_{i_0} X^{\nu_0} \dots \partial_{i_p} X^{\nu_p} \delta^{(D)}(x - X(\xi)) \end{aligned} \tag{8}$$

which can be rewritten as $\mathrm{d}(*e^{a\Phi}F) \sim \delta^{(d)}(y)$. So the presence of the source leads to the violation of the Bianchi identity of the magnetically dual field strength. In static gauge[5] these can be easily seen to be localized at $y = 0$. They match the singularities arising from $\Box_{\mathbb{R}^d} H$ if

$$\begin{aligned} h_d &= \frac{16\pi G_D T_p e^{-\frac{a}{2}\Phi_\infty} \Gamma(d/2)}{d(d-2)\pi^{d/2}} \frac{\Delta}{2(D-2)} \\ \frac{T_p}{\mu_p} &= \sqrt{2\frac{D-2}{\Delta}} e^{a\Phi_\infty} \\ b &= -\frac{a}{2} \end{aligned} \tag{9}$$

It is worthwhile to take a closer look at the equation of motion for the embedding fields $X^\mu(\xi)$. As no fields in our ansatz depend on the world-volume coordinates ξ^i, the Euler-Lagrange equations $\partial_i \frac{\partial \mathcal{L}}{\partial \partial_i X^\mu} - \frac{\partial \mathcal{L}}{\partial X^\mu}$ reduce to

$$\begin{aligned} \frac{\partial \mathcal{L}}{\partial X^\mu} &= \frac{T_p}{2} \sqrt{-\gamma} e^{b\Phi} [b \partial_\mu \Phi (\gamma^{ij} \partial_i X^\kappa \partial_j X^\lambda - p + 1) + \gamma^{ij} \partial_i X^\kappa \partial_j X^\lambda \partial_\mu g_{\kappa\lambda}] \\ &\quad - \frac{\mu_p}{(p+1)!} \epsilon^{i_0 \dots i_p} \partial_{i_0} X^{\mu_0} \dots \partial_{i_p} X^{\mu_p} \partial_\mu C_{\mu_0 \dots \mu_p} \end{aligned} \tag{10}$$

which vanishes identically for $\mu \in \{0, \dots, p\}$. Let us however generalize this part of the discussion to include non-extremal p-branes. That is, we assume the metric to take the form

$$\mathrm{d}s^2 = H_d(y)^{-2\frac{d-2}{\Delta}} [-f(y)\mathrm{d}t^2 + \mathrm{d}x_p^2] + \dots \tag{11}$$

[4] Note that some of the mathematical manipulations in the following section will be quite cavalier. Matching singularities, we will perform calculations with δ-functions without carefully regulating these.

[5] In static gauge

$$X^\mu(\xi^i) = \begin{cases} \xi^\mu & \mu \le p \\ 0 & \mu > p \end{cases}$$

where we dropped the transverse directions, which include off-diagonal elements in our choice of coordinates, but do not contribute to the following discussion. (10) reduces to

$$0 = [\frac{1}{2}a^2\frac{D-2}{\Delta} + \frac{(p+1)(d-2)}{\Delta}f]H_d^{-2}\partial_\mu H_d - \frac{p+1}{2}H_d^{-1}\partial_\mu f \tag{12}$$

Setting $f = 1 - \frac{Q}{y^{d-2}}$, it follows that the above is only solved if $Q = 0$, i.e. if the brane is extremal. Note further that in the non-extremal case, the term $H_d^{-1}\partial_\mu f$ diverges as $y \to 0$, while $H_d^{-2}\partial_\mu H_d \to 0$. One can interpret this behavior in the light of supersymmetry. By introducing f, we only modify the part of the brane action coupling to the metric, but not the one coupling to the $p+2$-form. The X^μ equation of motion can be thought of as a balancing between these two sectors (it imposes a relation between T_p and μ_p), so it is no surprise that it holds no longer once we have perturbed this balance. This might simply indicate an instability of the embedding or might indicate that it is not possible to find a source term for the non-extremal solution.[6] One should take in the account [13], where the authors constructed a finite temperature background including flavor-branes. In opposite to our discussion in the previous paragraph however, this background's non-extrmality is due to a horizon associated with the color-branes, while only the flavor-branes are represented by a source.

Dropping the 1 in the harmonic functions $H_d(y)$ in (5) leads generally to the near-horizon limit of the extremal p-brane considered. From (6) it follows however that the source-terms are still necessary in this limit – the argument does not depend on the asymptotic value of H_d.[7] Hence one can argue that to fully solve the equations of motion, one should add the source term for both the p-brane solutions as well as their near-horizon limit.

One should note that in this context, the introduction of the source term (7) did not add additional degrees of freedom to the background – γ_{ij} is auxiliary and $X^\mu(\xi)$ is merely a choice of gauge – but allowed us to lift the singularities as well as impose some relations between h_d, T_p, μ_p, G_D. In other words, by matching the source term with the p-brane solutions, we can give fix the charge and tension μ_p, T_p of the brane. Neither was the source term necessary to find the solution – we could have worked immediately with $S_{\text{grav}} + S_{\text{src}}$, but there was no need to do so.

3 Smeared sources

The situation changes a bit when we consider smeared sources. That is, the brane sources are not taken to be localized, but are continuously distributed over some open subset of space-time. Smearing was first seen in the context of T-duality and has today seen widepread use in the the field of gauge/string duality with a large number of flavors, $N_f \sim N_c$ ([2], [3], [15]- [23]). Here it simplifies the search for solutions as while also avoiding the problem of including corrections to the DBI- and Wess-Zumino terms appearing in the flavor action.

[6] The author is not aware of any general theorems regarding the existence of source terms for classical theories of gravity.

[7] After all, no matter whether in the near-horizon limit or not, H_d is harmonic everywhere except at the origin. See e.g. chapter 2.2 of [14].

To allow for smearing, we include a distribution density $\rho_{(d)}$ in the source term, which is formally a d-form on the space transverse to the additional branes. Introducing also a calibration form $\mathcal{K}_{(p+1)}$, which is essentially a volume form for the brane, one can then write the source term for *supersymmetric* sources as (see [19] for details)

$$S_{\rm src} = -T_p \int (e^{b\Phi}\mathcal{K}_{(p+1)} - C_{(p+1)}) \wedge \rho_{(d)} \tag{13}$$

Calculating the resulting equations of motion, the Maxwell equation takes the form

$$\mathrm{d}(*e^{a\Phi}F_{(p+2)}) = 16\pi G_D T_p \rho_{(d)} \tag{14}$$

– a straightforward generalization of (8). As a matter of fact, as long as there is some supersymmetry, it is sufficient to study the form-field equations such as (14) together with the supersymmetry conditions. The Einstein and Dilaton equations are then implied [24]. In contrast to the localized case of section 2, we would not have been able to derive suitable equations of motion without the source term, so in the context of smearing (over an open subset), the source term is essential.

4 D3D7 solutions

With all this in mind, let us take a look at D3-D7 solutions with 8 supercharges. This has previously been studied in [6] - [10] in the case where the D7-branes are localized. Note that the authors of [8] - [10] did not include any source terms in their actions working with $S = S_{\rm IIB}$, while [6] and [7] do include source terms for color- and flavor-branes. From our remarks in section 2 we suspect that this is not necessary (as their sources are localized), but we will see so explicitly. First, let us briefly summarize the background of [10] (in string frame):

$$\begin{aligned}
\mathrm{d}s^2 &= H^{-1/2}\mathrm{d}x^2 + H^{1/2}(\mathrm{d}z_1\mathrm{d}\bar{z}_1 + \mathrm{d}z_2\mathrm{d}\bar{z}_2 + e^{\Psi(z_3,\bar{z}_3)}\mathrm{d}z_3\mathrm{d}\bar{z}_3) \\
e^{\Psi(z_3,\bar{z}_3)} &= \tau_2(z_3)|\eta(\tau)|^4|z_3|^{-N_f/6} \\
\tau &= C_{(0)} + \imath e^{-\Phi} \\
F_{(5)} &= -\frac{1}{2\sqrt{2}(2\pi)^{7/2}g_s(\alpha')^2}(1+*)\mathrm{d}H^{-1}\wedge \mathrm{d}x^0 \wedge \cdots \wedge \mathrm{d}x^3
\end{aligned} \tag{15}$$

The complex structure (or axio-dilaton) τ is fixed by the presence of *localized* D7 branes. Crucial for us is that the warp factor $H(z_i, \bar{z}_i)$ must satisfy a deformation of the Laplace equation on the transverse space,

$$(\partial_1\bar{\partial}_1 + \partial_2\bar{\partial}_2 + e^{-\Psi}\partial_3\bar{\partial}_3)H = 0 \tag{16}$$

Strictly speaking, we are not interested in solutions to a modified Laplace equation, but a modified Poisson equation, as D3-branes will appear in a singularity, just as in the p-brane case

$$(\partial_1\bar{\partial}_1 + \partial_2\bar{\partial}_2 + e^{-\Psi}\partial_3\bar{\partial}_3)H = \delta^{(6)}(z) \tag{17}$$

In the following we will encounter different examples of H with different kinds of δ-functions appearing on the right hand side of equations like (17). To simplify

the notation, we shall always drop the δ-function, write the equations as (16) but keep in mind that H is usually singular at some point.

Looking for new solutions and working in the spirit of the flavoring program, we study $S = S_{\mathrm{IIB}} + S_{\mathrm{flavor}}$ with the source term being a superposition of D7 actions. Then we make the Ansatz (Einstein frame)

$$\begin{aligned}
\mathrm{d}s^2 &= e^{-\frac{1}{2}\Phi}[e^{2f}\mathrm{d}x_{1,3}^2 + e^{2g}\mathrm{d}v_4^2 + e^{2h}(\mathrm{d}w^2 + w^2\mathrm{d}\phi^2)] \\
F_{(5)} &= (1 + *_{10})(\mathrm{d}f_5 \wedge \mathrm{d}x^{0123}) \\
F_{(1)} &= f_1(w)w\mathrm{d}\phi \\
\Phi &= \Phi(v, w)
\end{aligned} \tag{18}$$

Where f, g, h, f_5, Φ depend on $w, v = \sqrt{v^i v^i}$ while f_1 depends on w alone. The most striking difference between (18) and (15) is that our choice for $F_{(1)}$ is in general not exact and can thus not be understood in terms of a 0-form potential $C_{(0)}$ and the relation $F_{(1)} = \mathrm{d}C_{(0)}$. In contrast, the appearance of $C_{(0)}$ in (15) implies $\mathrm{d}F_{(1)} = 0$, except at isolated singularities.[8] This is why the former ansatz will not allow for smeared D7 branes. Of course we study the action $S_{\mathrm{IIB}} + S_{\mathrm{src}}$, so there will be $\rho_{(2)}$ such that $\mathrm{d}F_{(1)} = \rho_{(2)}$. In other words, we will not need to impose a Bianchi identity for $F_{(1)}$, but are on the contrary rather interested in its explicit violation. Note also that our choice for $F_{(1)}$ implies that all D7 sources will be smeared along ϕ.

Demanding the existence of a SUSY spinor ϵ satisfying $\imath\Gamma^{0123}\epsilon = -\epsilon$ and $\Gamma^{4567}\epsilon = \epsilon$ we study the BPS-system given by

$$\begin{aligned}
0 &\overset{!}{=} \delta_\epsilon\lambda = \frac{1}{2}(\partial_\mu\Phi - \imath e^\Phi F_\mu)\Gamma^\mu\epsilon \\
0 &\overset{!}{=} \delta_\epsilon\psi_\mu = \partial_\mu\epsilon + \frac{1}{4}\omega_{\mu ab}\Gamma^{ab}\epsilon + \frac{\imath}{4}e^\Phi F_\mu + \frac{\imath}{16}\frac{1}{5!}F_{\nu\rho\sigma\tau\upsilon}\Gamma^{\nu\rho\sigma\tau\upsilon}\Gamma_\mu\epsilon
\end{aligned} \tag{19}$$

as well as the Bianchi identity for $F_{(5)}$. As we mentioned earlier, integrability ensures that the remaining equations of motion will be satisfied. One then sees quickly that any solution of the original ansatz can be rewritten in terms of only two functions, $H(v, w), \Delta_{gf}(w)$, and a set of integration constants

$$\begin{aligned}
\mathrm{d}s^2 &= e^{-\frac{c_\Phi}{2}}\{H^{-1/2}\mathrm{d}x_{1,3}^2 + H^{1/2}[\mathrm{d}v_4^2 + e^{-2(\Delta_{gf}-c_h)}(\mathrm{d}w^2 + w^2\mathrm{d}\phi^2)]\} \\
F_{(5)} &= (1+*)\mathrm{d}[(e^{-2c_\Phi}H^{-1} + c_{f_5})\mathrm{d}x^0 \wedge \mathrm{d}x^1 \wedge \mathrm{d}x^2 \wedge \mathrm{d}x^3] \\
F_{(1)} &= w(\partial_w e^{-2\Delta_{gf}})\mathrm{d}\phi \\
\Phi &= 2\Delta_{gf} + c_\Phi
\end{aligned} \tag{20}$$

[8] The non-exactness of $F_{(1)}$ explains also why in opposite to the earlier papers we do not rely on holomorphy of the axio-dilaton in the (w, ϕ) plane. If $F_{(1)}$ is exact, the supergravity variations can be phrased in terms of $C_{(0)}$ and the dilatino variation quickly takes the form of Cauchy-Riemann equations for $e^{-\Phi} + \imath C_{(0)}$.

subject to the modified Laplace/Poisson equation[9]

$$0 = \left[(\partial_v^2 + \frac{3}{v}\partial_v) + e^{\Delta_{gf}(w)-c_h}(\partial_w^2 + \frac{1}{w}\partial_w)\right] H(v,w) \tag{21}$$

which can be more succinctly summarized as

$$0 = (\Box_v + e^{\Delta_{gf}(w)-c_h}\Box_w)H(v,w) \tag{22}$$

Apart from the w and $z, \bar{z}$ dependence, this is the same equation as (16). However, while (15) was derived without use of an additional source term, the derivation of (21) was based on $S_{\mathrm{IIB}} + S_{\mathrm{D7}}$. As we found previously, as long as the sources are localized, one is free not to include the source term. Note that working in the spirit of gauge/string duality with flavor, we did not include a source term for the D3 *color* branes – yet of course, we could have. See footnote 9.

4.1 An aside: T-dualities

It is instructive to take a look at various T-dualities. There are two cases of interest – performing four T-dualities along the v^i, or performing two in the (w, ϕ) plane. In the latter case it is appropriate to change coordinates to Cartesian ones – (w^1, w^2) – to perform the dualities. The first case gives

$$\begin{aligned}
\mathrm{d}s^2 &= e^{-\frac{c_\Phi}{2}}\{e^{\Delta_{gf}}\mathrm{d}x_{1,3}^2 + e^{-\Delta_{gf}}[\mathrm{d}v_4^2 + e^{2c_h}H(\mathrm{d}w^2 + w^2\mathrm{d}\phi^2)]\} \\
\Phi &= c_\Phi - \log H \\
F_{(5)} &= (1+*)\mathrm{d}(e^{2\Delta_{gf}}\mathrm{d}x^0\wedge\mathrm{d}x^1\wedge\mathrm{d}x^2\wedge\mathrm{d}x^3) \\
F_{(1)} &= -e^{-2c_\Phi}\partial_w(e^{-2c_\Phi}H + c_{f_5})w\mathrm{d}\phi
\end{aligned} \tag{23}$$

Comparing (20) and (23) shows the result of the dualitites to be a swap $-2\Delta_{gf} \leftrightarrow \log H$. Now note that while the Buscher rules for T-dualities in supergravity only apply for v^i to be an isometry of the background, i.e. for $\partial_{v^i}H = 0$, the substitution $-2\Delta_{gf} \leftrightarrow \log H$ is valid at the level of the BPS equations and equations of motion too. The simple reason is that the BPS equations are all trivially satisfied when written in terms of Δ_{gf} and H, the equation of motion for $F_{(1)}$ is always satisfied as well as $F_{(1)}$ depends only on $\mathrm{d}\phi$, so the only points of interest are the Bianchi identities for $F_{(5)}$ and $F_{(1)}$. These however do not need to be satisfied if we allow for smeared brane sources. Again we point out that we only included an explicit source term for the D7-branes, that have now been turned into D3s.

T-dualities along w^1, w^2 lead to

$$\begin{aligned}
\mathrm{d}s^2 &= e^{-\frac{\Delta_{gf}}{2}-\frac{c_\Phi}{2}+c_h}[H^{-1/4}(\mathrm{d}x_{1,3}^2 + e^{-2c_h}\mathrm{d}w_2^2) + H^{3/4}\mathrm{d}v_4^2] \\
\Phi &= 3\Delta_{gf} + c_\Phi - 2c_h - \frac{1}{2}\log H \\
F_{(7)} &= \mathrm{d}[(e^{-2c_\Phi}H^{-1} + c_{f_5})w\mathrm{d}x^0\wedge\cdots\wedge\mathrm{d}x^3\wedge\mathrm{d}w\wedge\mathrm{d}\phi] \\
F_{(1)} &= -\mathrm{d}(e^{-2\Delta_{gf}(w)})
\end{aligned} \tag{24}$$

[9] Crucially, (21) arises from the Bianchi identity on $\mathrm{d}F_{(5)} = 0$. As we have seen in sections 2 and 3, these identities relate directly to the presence of sources and should be rewritten as $\mathrm{d}F_{(5)} = \rho_{(6)}$ as we are looking for backgrounds with D3 sources. So strictly speaking, there should be a source density on the right hand side of (21), at least a δ-function. As we are looking for smeared D7 branes in backgrounds with localized D3s, we ignore this distinction and just keep in mind that when solving (21), we are looking for solutions that show singular behavior at $(v, w) = (0, 0)$.

For $\Delta_{gf} = 0$, (23) and (24) reduce to the standard flat D7 and D5 solutions. This is of course expected, as (18) describes a stack of D3-branes in flat space. Turning on Δ_{gf} adds five- and one-form flux to the T-dual backgrounds respectively; for (24) the one-form flux is exact, however, so there are only additional D7 sources if $\Delta_{gf}(w)$ is not differentiable at isolated points. We are dealing with a D7-D3 and a D5(-D7) system, respectively.

In the context of gauge/string duality, the T-dualities along the v^i should be of interest. After all, it exchanges the N_c color D3-branes with the N_f flavor D7-branes – at first glance, we have a duality $(N_c, N_f) \leftrightarrow (N_f, N_c)$. Of course the precise form of the duality depends on the brane distributions.

4.2 Simple, known solutions

For $\Delta_{gf} = 0, c_h = 0$, there is of course the standard D3-brane solution,

$$H_3 = 1 + \frac{r_3^4}{(v^2 + w^2)^2} \tag{25}$$

the laplacian of which has a δ-function singularity at $(v, w) = (0, 0)$ due to the presence of the D3-branes. The near horizon limit is given by

$$H_3 \mapsto \frac{r_3^4}{(v^2 + w^2)^2} \tag{26}$$

There are further solution that depend on only one variable and have thus additional isometries in the background

$$\begin{aligned} H(v, w) &= 1 + \frac{r_5^2}{v^2} \\ H(v, w) &= 1 + r_7 \log w \end{aligned} \tag{27}$$

They are the harmonic functions in four and two dimensions respectively.[10] They are singular at $v = 0$ or $w = 0$. The standard interpretation here is to think of the D3-branes as having been smeared over (w, ϕ) or the v^i. I.e. the smeared branes are now codimensions four or codimension two objects. Performing two (four) T-dualities along the additional isometries leads to the standard D5 (D7) solutions. Remeber that (21) is linear, so any superposition of (25) and (27) is a solution as well.

4.3 Analytic solutions

Looking for new solutions of (21), we will make use of the fact that there are not cross derivative terms of the form $\partial_v \partial_w$. Hence the PDE is separable and we may look for solutions of the form

$$\begin{aligned} H(v, w) &= H_v^\times(v) \times H_w^\times(w) \\ H(v, w) &= H_v^+(v) + H_w^+(w) \end{aligned} \tag{28}$$

[10] If one wonders why (21) is not symmetric under $v \leftrightarrow w$ for $\Delta_{gf} = c_h$, the explanation can be found here. v and w are the radial coordinate in spaces of different dimension.

after which (21) takes the form

$$
\begin{aligned}
0 &= H_w^\times[\frac{3}{v}(H_v^\times)' + (H_v^\times)''] + e^{\Delta_{gf}-c_h}H_v^\times[\frac{1}{w}(H_w^\times)' + (H_w^\times)''] \\
0 &= [\frac{3}{v}(H_v^+)' + (H_v^+)''] + e^{\Delta_{gf}-c_h}[\frac{1}{w}(H_w^+)' + (H_w^+)'']
\end{aligned}
\tag{29}
$$

The crucial point is that, independently of $\Delta_{gf}(w)$ and c_h, any solution of the ODEs

$$
\begin{aligned}
H_v''(v) &= -\frac{3}{v}H_v'(v) \\
H_w''(w) &= -\frac{1}{w}H_w'(w)
\end{aligned}
\tag{30}
$$

gives a solution of type IIB supergravity. Of course, finding an analytic solution to (30) is quite straightforward. As a matter of fact, these are the harmonic functions of (27)

$$
\begin{aligned}
H_v &= \frac{c_{v1}}{v^2} + c_{v2} \\
H_w &= c_{w2}\log w + c_{w3}
\end{aligned}
\tag{31}
$$

with $c_{v1}, c_{v2}, c_{w2}, c_{w3} \in \mathbb{R}$. And so we have two new families of analytic solutions

$$
\begin{aligned}
H^\times(v,w) &= (c_{w2}\log w + c_{w3})(\frac{c_{v1}}{v^2} + c_{v2}) \\
H^+(v,w) &= c_{w2}\log w + \frac{c_{v1}}{v^2} + c_{v2} + c_{w3}
\end{aligned}
\tag{32}
$$

Of course these are just (27) and one might ask what is new. The point is that (32) hold together with (20) for arbitrary $\Delta_{gf}(w)$, and hence for arbitrary D7-brane distributions. The interpretation of these solutions is similar to that given at the end of section 4.2. The D3-branes are smeared over some of their transverse directions, but now also accomodate for any D7 distribution imposed by choice of $\Delta_{gf}(w)$.

It is interesting to note that (27) reappear as (32) independently of whether we add D7-sources or not. Of course, it would be much more interesting to find the equivalent of (25) in the presence of $\Delta_{gf} \neq 0$. We shall do so in section 4.5 numerically.

4.4 Brane distributions

At this point we will take a look at a few brane distributions. Note that

$$
\begin{aligned}
\rho_{(2)} &= [(\partial_w^2 + \frac{1}{w}\partial_w)e^{-2\Delta_{gf}}]w\mathrm{d}w \wedge \mathrm{d}\phi \\
&= (\Box_w e^{-2\Delta_{gf}(w)})w\mathrm{d}w \wedge \mathrm{d}\phi
\end{aligned}
\tag{33}
$$

From out ansatz it follows that D7-branes are always smeared along ϕ, so the simplest distribution is a δ-function one in the w direction,

$$
\rho_{(2)} = Q\delta(w - w_0)w\mathrm{d}w \wedge \mathrm{d}\phi \tag{34}
$$

where Q is some normalization constant. We can integrate the resulting flux,

$$
\int_{S^1} F_{(1)} = 2\pi Q w_0 \theta(w - w_0) \tag{35}
$$

and it follows that

$$Q = \frac{N_f}{2\pi w_0} \tag{36}$$

Then

$$e^{-2\Delta_{gf}} = \frac{N_f}{2\pi w_0}[c_1 \log w + w_0 \theta(w - w0) \log \frac{w}{w0} + c_{\mathrm{src}}] \tag{37}$$

Naturally this should be positive for all values of $w \in \mathbb{R}^+$, hence it seems appropriate to set $c_1 = 0$. Also, as both $e^{-\Delta_{gf}}$ and $e^{\Delta_{gf}}$ appear in the metric, $e^{-2\Delta_{gf}} \geq 0$ is a good assumption that is guaranteed by fixing $c_{\mathrm{src}} > 0$. Furthermore, our numerical studies in section 4.5 will show that varying c_{src} does influence the form of the solutions rather strongly. To avoid this, we will fix it to $c_{\mathrm{src}} = Q^{-1}$ so that the constant term in $e^{-2\Delta_{gf}}$ does not vary with N_f.

A similarly interesting case is given by

$$\begin{aligned} \rho_{(2)} &= Q\theta(w - w_0) w \mathrm{d}w \wedge \mathrm{d}\phi \\ e^{-2\Delta_{gf}} &= Q[c_1 \log w + \frac{1}{4}\theta(w - w_0)(w^2 - w_0^2 - 2w_0^2 \log \frac{w}{w_0}) + c_{\mathrm{src}}] \end{aligned} \tag{38}$$

For the same reasons as above we fix $c_1 = 0$ and $c_{\mathrm{src}} = Q^{-1}$. Concerning the normalization, we have

$$F_{(1)} = \frac{Q}{2}(w^2 - w_0^2)\theta(w - w_0)\mathrm{d}\phi \tag{39}$$

leading to a radially dependent charge

$$N_f(w) = Q\pi(w^2 - w_0^2) \tag{40}$$

The fact that $N_f(w)$ behaves like a two-dimensional area is no accident. After all, we assume a homogeneous brane distribution in the (w, ϕ) plane for all $w \geq w_0$.

4.5 Numeric solutions

Let us now take a look at numeric solutions of (21). We are dealing with a deformation of the Laplace (Poisson) equation, that is, a homogeneous, elliptic, separable PDE of second order, and use the Fortran package Mudpack[11] to do so. Our aim is to perform a qualitative study of deformations of the original $AdS_5 \times S^5$ solution (25) that includes additional D7-branes. We fix the parameter $r_3 = 1$ and solve the equation in a rectangular domain in the (v, w) plane specified by

$$0.2 \leq v, w \leq 2.6 \tag{41}$$

on a 129×129 grid. Some experimentation shows that one obtains a good agreement with the analytic solutions in the absence of D7 branes when imposing the Neumann boundary conditions at $w = 0.2$ and $w = 2.6$ and Dirichlet ones at $v = 0.2$ and $v = 2.6$. I.e.

$$\begin{aligned} H &= \frac{1}{(v^2 + w^2)^2} \qquad \text{at } w = 0.2 \vee w = 2.6 \\ \partial_v H &= -\frac{4v}{(v^2 + w^2)^3} \qquad \text{at } v = 0.2 \vee v = 2.6 \end{aligned} \tag{42}$$

[11] Mudpack can be found at http://www.cisl.ucar.edu/css/software/mudpack/.

However, the physical significance of the boundary conditions is not entirely clear and it might be appropriate to modify the boundary conditions when changing the source density Δ_{gf}.

Figure 1 shows the analytic solution $H_3 = \frac{1}{(v^2+w^2)^2}$. Our numeric solution for $e^{-\Delta_{gf}} = 1$ (not shown) agrees up to $\Delta H = \pm 0.0001$. We then proceed

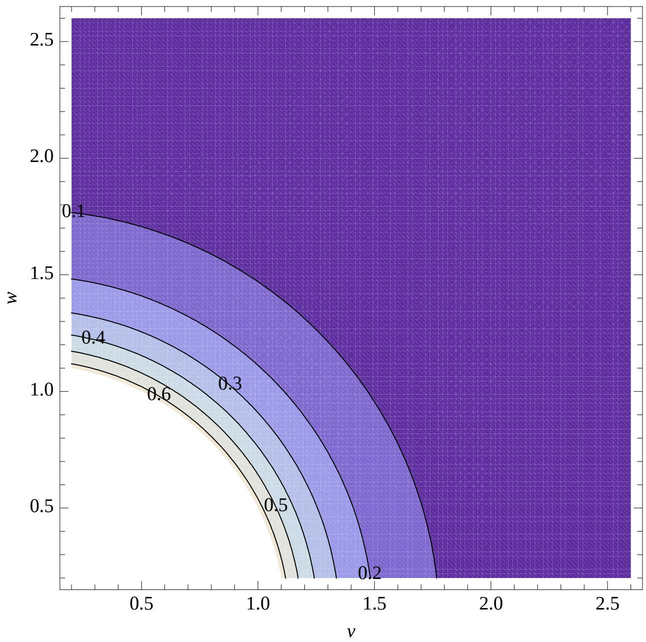

Figure 1: Plot of the analytic solution $H_3 = (v^2 + w^2)^{-2}$. There is a singularity in H_3 at the origin charactereistic to the presence of D3 branes

to include D7 branes via changing $e^{-2\Delta_{gf}}$. In all these cases we approximate Heaviside θ functions by

$$\begin{aligned} \theta(w) &= \frac{1}{2} + \frac{1}{2}\tanh(kw) \\ k &= 2.5 \end{aligned} \tag{43}$$

Larger values of k make for a sharper transition, in the case $k = 2.5$ we have $1 - \theta(0.5) \sim 0.0758$. Figure 2 shows the case $e^{-2\Delta_{gf}} = \theta(w-1)\log(w) + 1$ while figure 3 uses $e^{-2\Delta_{gf}} = 10[\theta(w-1)\log(w)] + 1$. So in each case, there is a stack of D7 branes localized at $w = 1$, yet smeared along ϕ. The changes in the solutions are not drastic, but differ from H_3 by one or two orders of magnitude, so instead of plotting H for each case, we show the difference to the pure D3-brane solution of figure 1, $H - H_3$.

Things change considerably when we scale the source density by another factor of 50, i.e. we set $e^{-2\Delta_{gf}} = 500[\theta(w-1)\log(w)+]1$ (fig. 4). One can see quite clearly that the background is dominated by the D7 branes extending along the v^i while the boundary conditions, especially at $(0.2, 0.2)$ are still those of the D3 background.

Figure 5 shows a brane distribution along the lines of (38). That is, the number of flavors runs with w^2. Of course, here the UV should be dominated

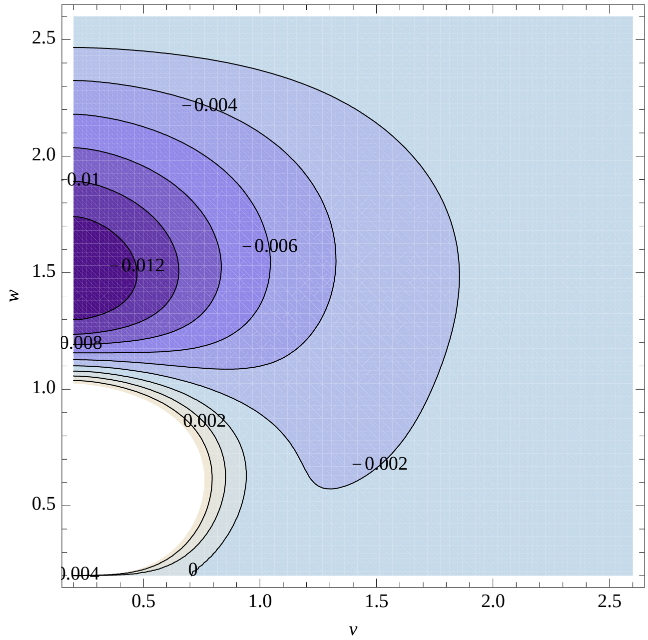

Figure 2: ΔH for $e^{-2\Delta_{gf}} = \theta(w-1)\log(w) + 1$.

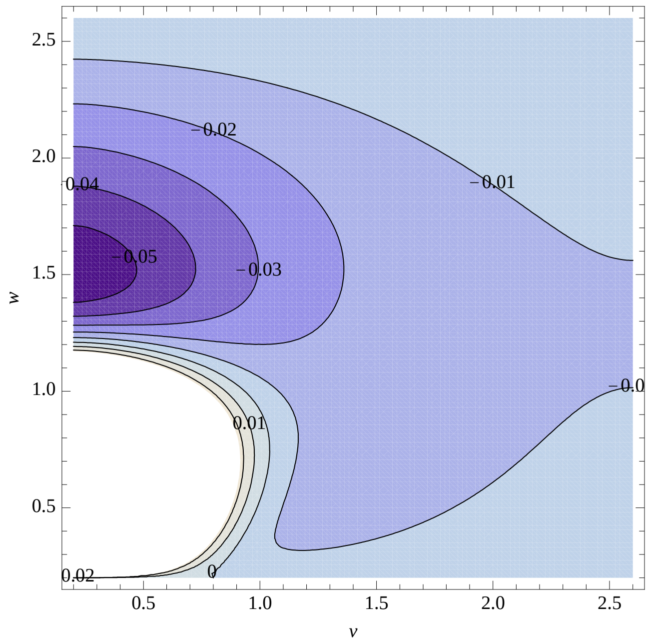

Figure 3: ΔH for $e^{-2\Delta_{gf}} = 10[\theta(w-1)\log(w)] + 1$.

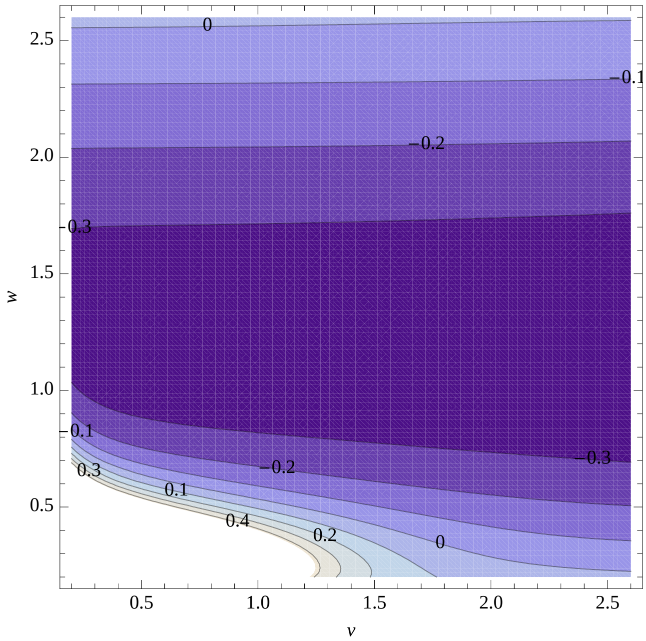

Figure 4: H for $e^{-2\Delta_{gf}} = 500[\theta(w-1)\log(w)] + 1$.

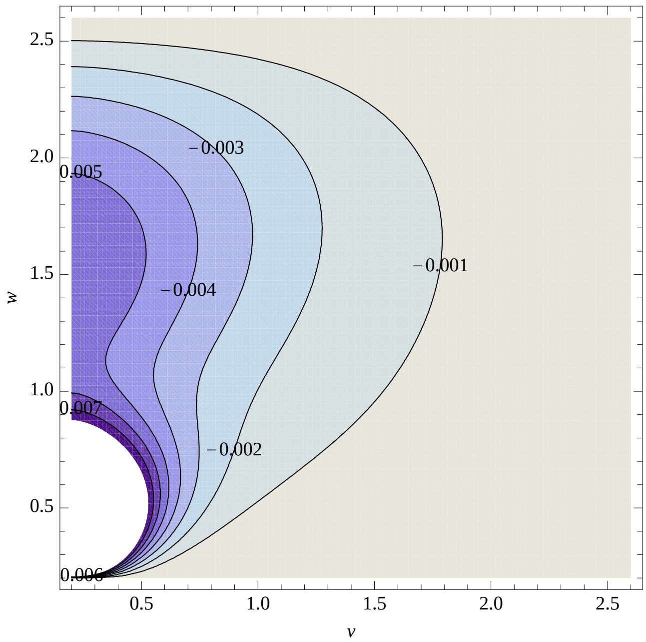

Figure 5: ΔH for $e^{-2\Delta_{gf}} = \frac{1}{4}\theta(w-1)[w^2 - 1 - 2w^2\log(w)] + 1$.

by increasing number of D7 branes and it might be appropriate to adjust the boundary conditions at $v = 2.6$ and $w = 2.6$. Based on the T-dual of the analytic D7 solution, we set them to

$$\begin{aligned} H &= \log w \quad \text{at } w = 2.6 \\ \partial_v H &= \frac{1}{w} \qquad \text{at } v = 2.6 \end{aligned} \tag{44}$$

while those at $v = 0.2$ and $w = 0.2$ remain as in (42). The result is shown in 6. Note that having changed the boundary conditions, the solution is quite different to 1 and we plot H instead of ΔH.

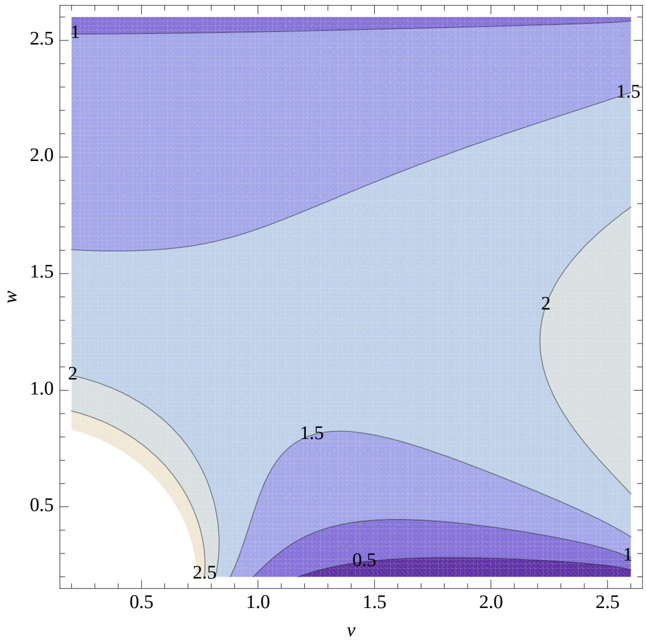

Figure 6: H for $e^{-2\Delta_{gf}}$ as in fig 5, however, at $v = 2.6$ and $w = 2.6$ we imposed the boundary conditions characteristic for D3 branes smeared along v^i – a system T-dual to D7 branes.

5 Conclusions

In this paper, we have analyzed (1) from several perspectives. From the perspective of the p-brane action in sections 2 and 3, we realized that color- and flavor-branes are actually on a very similar footing. In principle one should include source terms for both as was done in [7], so this observation is not new, as we mentioned before. However the presence of source terms is only necessary if the associated sources are to be smeared over an open subset of space-time, which is why the sourc-term is essential for the flavor-branes that are usually assumed to be smeared.

The impression that color and flavor-branes can – from the supergravity perspective – be taken to be on an equal footing was again confirmed by our

observations in section 4.1, where we were able to exchange color and flavor-branes by performing four T-dualities in the directions transverse to the D3s. Curiously, we had included explicit source terms for the (smeared) flavor D7-branes while not doing so for their localized[12] cousins. One should also keep in mind that [10] obtained highly similar results working with the supergravity action alone – while including suitable δ-function sources, of course.

We also raised the issue whether it is generally possible to find a source-term for a given solution – especially in cases where supersymmetry is broken. As discussed in [11], the problem lies in the fact that for sources in theories of gravity, the energy of the source is not localized at the source but also stored in the self-energy of the surrounding gravitational field. Only in the presence of supersymmetry, where gravitational effects are canceled by those of a different field – the Maxwell-type p-form fields in this case – can one find a suitable source term. Again we point out [13] however, where the authors have constructed a finite-temperature background including additional flavor terms.

Naturally our comments and observations made here are only valid for the examples studied, and it would be interesting to study the issue of source terms for color-branes for more complex supergravity backgrounds dual to confining gauge theories, such as [4], [12] and [25]. From the point of gauge/string duality, the crucial point is there whether there are open string states in the spectrum, that should only appear in non-confining theories. In other words, one expects that for confining backgrounds it should not be possible to find source terms for the color branes. Verifying this explicitly would be an avenue item for future research.

Finally, we found a series of new D3-D7 backgrounds with smeared D7-branes. Here, the analytic solutions captured in (32) have the interesting property that for any distribution of D7-branes encoded in Δ_{gf}, the D3s distribute themselves such there is a solution. It would be intersting to interpret the solutions of sections 4.3 and 4.5 in the context of gauge/string duality, another problem that we leave for future work.

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